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Recombination of intersecting D-branes and cosmological inflation

Marta Gómez-Reino and Ivonne Zavala C.

Centre for Mathematical Sciences, DAMTP, University of Cambridge
Cambridge, CB3 0WA, UK
E-mail: mgr25@damtp.cam.ac.uk, eiz20@damtp.cam.ac.uk

ABSTRACT: We consider the interactions between Dp-branes intersecting at an arbitrary number of angles in the context of type-II string theory. For cosmology purposes we concentrate in the theory on $\mathbb{R}^{3,1} \times T^6$. Interpreting the distance between the branes as the inflaton field, the branes can intersect at most at two angles on the compact space. If the configuration is non supersymmetric we will have an interbrane potential that provides an effective cosmological inflationary epoch at the four-dimensional intersection between the branes. The end of inflation occurs when the interbrane distance becomes small compared with the string scale, where a tachyon develops triggering the recombination of the branes. We study this recombination due to tachyon instabilities and we find the possibility for the final configuration to be again branes intersecting at two angles. This preserves the interesting features that are present in the intersecting brane models from the string model building point of view also after the end of inflation. This fact was not present in the models of branes intersecting at just one angle. This kind of recombination can be also important in other string contexts.

KEYWORDS: D-branes, Cosmology of Theories beyond the SM, Physics of the Early Universe.

Contents

1. Introduction

Inflationary cosmology [1] continues being the most likely scenario that provides an explanation to many of the problems in standard cosmology, such as the flatness, the horizon, or the monopole problems. However, it is still a successful idea seeking for a successful theoretical realization. In this sense, it is natural to look for such a theoretical realization within string theory, since it can provide us with a quantum description of gravity at high energies. In fact, there has been recently a lot of work towards a consistent realization of this scenario using different brane configurations which appear naturally in string theory [2]–[12]. In these models the inflaton potential has been identified with one of the moduli potentials either coming from the interactions between the branes or from the complex structure of the compact space.

Systems with interacting branes in non-supersymmetric configurations seem to be natural candidates for inflation. One of the reasons for this is that their interactions are well known within string theory (see for example [15]) so one can calculate the attractive potential between the branes. Another reason is that, since the configurations are unstable — as they are non supersymmetric — tachyons will in general appear opening the possibility of a hybrid inflationary [17] regime which would naturally trigger the end of inflation. This possibility was first described in [3].

Of particular interest are the models with brane configurations where the supersymmetry is only slightly broken [6, 7, 10, 11]. In particular, within this class of models, configurations with branes intersecting at angles are specially attractive. The main reason is that chiral fermions appear naturally at the intersections between the branes [13]. This

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property has been exploited widely in the construction of (semi) realistic brane models with a spectrum very close [18]–[24] to that of the standard model of particle physics. There are also some supersymmetric versions of these intersecting brane models [25].

Within the context of intersecting brane models, in ref.[11] it was shown that it is possible to obtain a successful inflationary period from slightly non-supersymmetric configurations consisting of D4-branes intersecting at one angle on a six-dimensional compact space. Since the supersymmetry was broken that gave rise to an interaction potential between the branes parametrized by the interbrane distance, which was identified with the inflaton field. Inflation ended when the distance between the branes became of order of the string scale. At that moment a tachyon appeared triggering the recombination of the branes to a single brane stable configuration, and then losing the chirality property at the intersection present at the beginning of inflation.

Following this line, we first consider the most general configuration of branes making n angles in type-II string theory. We then concentrate on the theory on $\mathbb{R}^{3,1} \times T^6$. In that case we study the most general angled configuration that allows for an interbrane potential in terms of the separation between the branes — on the compact space — and that can give rise to an inflationary period due to this interbrane distance (in the effective four-dimensional theory). As we will see, this requirement implies that the maximal number of angles that the branes can make on the compact space is $n = 2$. As will be explained below, a successful inflationary epoch will occur as long as the starting configuration is not far away from the supersymmetric one. An interesting feature of these configurations, compared with the one angled D-brane case, is the fact that branes intersecting at two angles in the compact space open the possibility of ending up, after the recombination of the branes (that is, after the end of inflation), with a final state being a supersymmetric configuration with again two branes intersecting at angles. This is the most attractive property of these configurations since they would provide a first step towards the construction of semi realistic (supersymmetric) brane models with chiral matter after the end of inflation in the context of intersecting brane models (the possibility of obtaining a realistic model after inflation was already considered in [8] in the context of brane-antibrane interactions at orbifold singularities). This possibility was not present in the case of branes intersecting at just one angle.

The structure of the paper is as follows: in section 2 we explain the general type-II string brane interactions between branes at angles. Then we concentrate in the two angled case and at the end we discuss the different possibilities for the recombination of the system that appear in this case. In section 3 we review the basic inflationary parameters and concentrate in the two specific brane models which can give rise to a successful inflationary epoch. In section 4 we give some comments about the end of inflation and in the last section we will conclude.

2. The string model

This section is divided in three parts. In the first one, we review the interaction potential that appears between Dp-branes intersecting each other at different angles. In the second subsection we explain the string model that we are interested in, and also the approximations that we will consider for a suitable application to inflation. Finally, in the last subsection we analyse the decay and recombination of the system due to tachyon instabilities.

2.1 Interactions between intersecting D-branes

Let us consider two Dp-branes in type-II string theory, intersecting at n angles inside the ten-dimensional space. We will review in this subsection the interaction potential arising when the two Dp-branes are located in such a way that they intersect at a number n of angles, $\Delta\theta_1, \dots \Delta\theta_n$, being the remaining $(p+1-n)$ dimensions of the branes either parallel to each other or common to both branes.

The interaction between the branes can be computed from the exchange of massless closed string modes. This can be computed from the one-loop vacuum amplitude for the open strings stretched between the two Dp-branes, that is given by

$$\mathcal{A} = 2\int \frac{dt}{2t}\,\mathrm{Tr}\, e^{-tH}\,, \tag{2.1}$$

where H is the open string hamiltonian. For two Dp-branes making n angles in ten dimensions this amplitude can be computed to give [14, 15][1]

$$\mathcal{A} = V_p \int_0^\infty \frac{dt}{t} \exp^{-\frac{t\,Y^2}{2\pi^2\alpha'}} (8\pi^2\alpha' t)^{-\frac{p-3}{2}} \left(-iL\eta(i\,t)^{-3}(8\pi\alpha' t)^{-1/2}\right)^{4-n} (Z_{NS} - Z_R)\,, \tag{2.2}$$

with

$$\begin{aligned} Z_{NS} &= (\Theta_3(0\mid it))^{4-n}\prod_{j=1}^{n}\frac{\Theta_3(i\Delta\theta_j t\mid it)}{\Theta_1(i\Delta\theta_j t\mid it)} - (\Theta_4(0\mid it))^{4-n}\prod_{j=1}^{n}\frac{\Theta_4(i\Delta\theta_j t\mid it)}{\Theta_1(i\Delta\theta_j t\mid it)}\,,\\ Z_R &= (\Theta_2(0\mid it))^{4-n}\prod_{j=1}^{n}\frac{\Theta_2(i\Delta\theta_j t\mid it)}{\Theta_1(i\Delta\theta_j t\mid it)}\,, \end{aligned} \tag{2.3}$$

being the contributions coming from the NS and R sectors. Also in (2.3) Θ_i are the usual Jacobi functions and η is the Dedekin function. Furthermore, in (2.2) by Y we mean the distance between both branes, $Y = \sqrt{\sum_k Y_k^2}$, where k labels the dimensions in which the branes are separated and Y_k the distance between both branes along the k direction.

The various terms in the expression (2.2) can be understood as follows: a general Dp-brane in ten dimensions can make at most $\min(p, 9-p)$ angles, e.g. a D3-brane can make at most 3 angles, and a D7-brane can make at most 2 angles, etc. Each time that one angle is taken to zero, a factor of $-iL\eta(i\,t)^{-3}(8\pi\alpha' t)^{-1/2}$ appears in the amplitude, reflecting the fact that the branes become parallel in one dimension, where $V_p L^{4-n}$ gives the volume of the common dimensions to both branes.

We are interested now in the small t limit of (2.2), that is, the large distance limit ($Y \gg l_s$). This is the right limit that takes into account the contributions coming from the massless closed strings exchanged between the branes. Using the well known modular

[1] Configurations with more than two branes intersecting with each other at angles, in only one plane, were considered in [16].

properties of the Θ and η functions we obtain, in the $t \to 0$ limit, that the amplitude is just given by

$$\mathcal{A}(Y, \Delta\theta_j) = \frac{V_p L^{4-n} F(\Delta\theta_j)}{2^{p-2}(2\pi^2\alpha')^{(p+1-n)/2}} \int t^{-\frac{p+n-5}{2}} \exp^{-\frac{tY^2}{2\pi^2\alpha'}} dt\,, \tag{2.4}$$

where the function F contains the dependence on the relative angles between the branes, and is extracted from the small t limit of (2.3). The exact form of this function is given by

$$F(\Delta\theta_j) = \frac{(4-n) + \sum_{j+1}^{n} \cos 2\Delta\theta_j - 4\prod_{j=1}^{n} \cos\Delta\theta_j}{2\prod_{j=1}^{n} \sin\Delta\theta_j}\,. \tag{2.5}$$

The relation between the angles $\Delta\theta_j$ that make this function F vanish, correspond to the supersymmetric configurations of the two brane system for each given number of angles. For example it is easy to see that for branes at just one angle the function (2.5) only vanishes when $\Delta\theta_1 = 0$, corresponding to the supersymmetric situation. As we will see later, in the two angled case the function (2.5) vanishes when $\Delta\theta_1 = \pm\Delta\theta_2$ [14].

The interaction potential between the branes can then be calculated by performing the integral (2.4). This integral is just given in terms of the Euler Γ-function, so the potential has the following form

$$V(Y, \Delta\theta_j) = -\frac{V_p L^{4-n} F(\Delta\theta_j)}{2^{p-2}(2\pi^2\alpha')^{p-3}} \Gamma\left(\frac{7-p-n}{2}\right) Y^{(p+n-7)}\,. \tag{2.6}$$

Note that for $p + n = 7$ this expression is not valid as $\Gamma(0)$ is not a well defined function. In fact in that case the integral (2.4) is divergent, so we need to introduce a lower cutoff to perform it. If we denote by Λ_c the cutoff, the integral becomes

$$V(Y, \Delta\theta_j) = \frac{V_p L^{p-3} F(\Delta\theta_j)}{(4\pi^2\alpha')^{p-3}} \ln\frac{Y}{\Lambda_c}\,. \tag{2.7}$$

Taking a quick look at the form of the potentials (2.6), (2.7), it is natural to find them interesting from the cosmological point of view. In fact if one identifies the distance Y between the branes as the inflaton field, it is clear that the potential can be made flat enough by choosing appropriately the angles. This can be seen straight forwardly by noticing that the function F can be made very small as one approaches to a supersymmetric configuration. Also it is interesting to check that the potentials (2.6), (2.7) are always attractive since the Γ-function changes sign when its argument becomes negative.

Now, we are interested in inflationary cosmology in four dimensions coming from this configurations, so we concentrate in models where both branes span four non-compact dimensions and intersect over the remaining six dimensions that we take to be compactified. We then want to identify the inflationary field with the interbrane distance on the compact six-dimensional space. Since both branes are extended and parallel in the four non-compact dimensions, the maximum number of angles that they can make, having an interbrane distance different from zero, is two. That is, the most general situation to consider in such configurations is take $n = 2$ in (2.6). The situation with just one angle can then be extracted from the two angled case. In fact that situation was considered in [11, 12] so we will concentrate here only in the case of branes intersecting at two angles. The precise model under study will be discussed in detail in the following subsection.

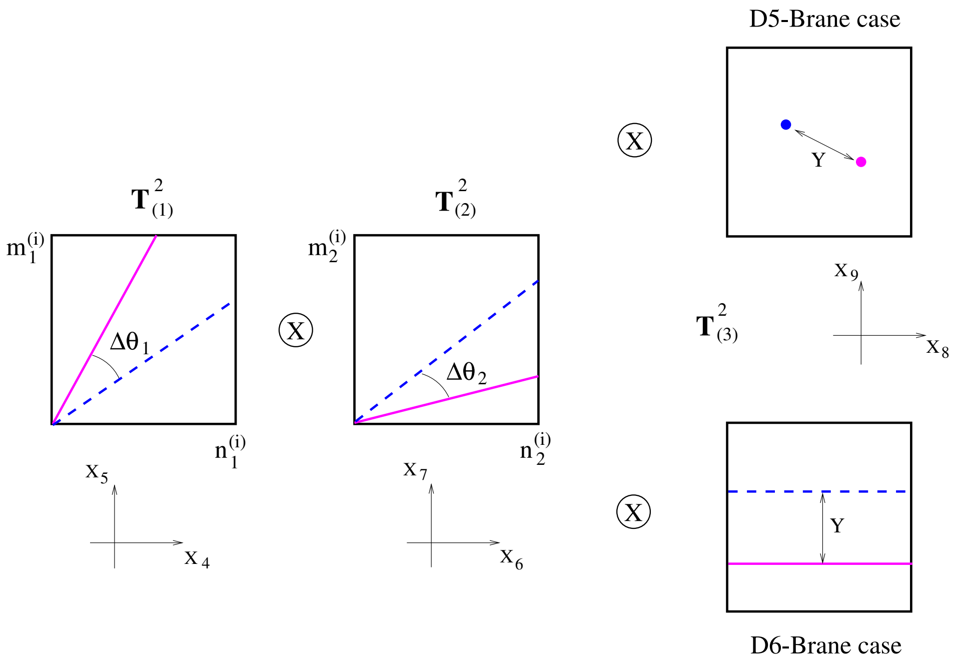

Figure 1: Diagram of the configurations we are considering in the two cases discussed.

2.2 Description of the model

Let us then consider now two Dp-branes in type-II string theory spanning (3+1) dimensions along the non-compact space $\mathbb{R}^{3,1}$, with their remaining spatial dimensions laying on the compact space, that is taken to be a factorisable six torus, i.e. $T^6 = T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}$. As explained above, since we want to have a non-zero interbrane separation Y, the most general possibility that we can consider is take D5 or D6-branes making two angles on the compact space. We discuss now this situation.

As it is shown in figure 1 we will consider that the branes wrap some factorisable 2-cycles along two of the tori of the compact space, $T^2_{(1)} \times T^2_{(2)}$, and they are separated by a distance $Y = \sqrt{\sum_k Y_k^2}$ on the third torus $T^2_{(3)}$. We will assume for simplicity that we have squared tori with radii R, and that each Dp_i-brane wraps a factorisable cycle with homological charge $(n_I^{(i)}, m_I^{(i)})$ on the I'-th torus. This means that each brane wraps a number $(n_I^{(i)}, m_I^{(i)})$ of times over the corresponding homology cycle of the I'-th torus.

The angle between the branes for squared tori, as can be seen in figure 1, is given in terms of the homological charges of the branes on each torus by the formula

$$\theta_I^{(i)} = \arctan \frac{m_I^{(i)}}{n_I^{(i)}}, \qquad I = 1, 2, \qquad \Delta\theta_I = \left|\theta_I^{(1)} - \theta_I^{(2)}\right| . \tag{2.8}$$

Note that if both branes are wrapped on the same cycles and with the same orientations in each tori (i.e. $\Delta\theta_1 = \Delta\theta_2 = 0$ or $\Delta\theta_1 = \Delta\theta_2 = \pi$) that means that they are parallel. In this case we have an $\mathcal{N} = 4$ supersymmetric theory from the four-dimensional point of view, that is, we have a configuration that preserves sixteen supercharges. If they wrap the same cycles but with opposite orientations along one of the two tori and the same orientation on the other (i.e. $\Delta\theta_1 = 0, \Delta\theta_2 = \pi$ or vice versa) we have a brane-antibrane configuration, which breaks completely the supersymmetry.

In a generic case with topologically different cycles we will have a configuration where the branes intersect at non-zero relative angles $\Delta\theta_I$ in the range $0 \leq \Delta\theta_I \leq \pi$, $I = 1, 2$. This configuration is supersymmetric when $|\Delta\theta_1| = |\Delta\theta_2|$, and it preserves 1/4 of the supersymmetries (that is, we will have a $\mathcal{N} = 1$ supersymmetric theory from the four-dimensional point of view) [14]. For this configuration the total homology class of the 2-cycles is the sum of the homology classes of the two branes, being — in the supersymmetric case — a special lagrangian in the complex structure. In the general case, that is, when $|\Delta\theta_1| \neq |\Delta\theta_2|$, all the supersymmetries are broken.

In this non-supersymmetric case (considering that $\Delta\theta_I \in [0, \pi]$) the mass of the lightest scalars is controlled by the angles between the Dp-branes and is given by [13, 14]

$$\begin{aligned} \alpha' M_1^2 &= \frac{Y^2}{4\pi^2\alpha'} + \frac{1}{2}(\Delta\theta_1 - \Delta\theta_2)\,, \\ \alpha' M_2^2 &= \frac{Y^2}{4\pi^2\alpha'} + \frac{1}{2}(\Delta\theta_2 - \Delta\theta_1)\,, \\ \alpha' M_3^2 &= \frac{Y^2}{4\pi^2\alpha'} + \frac{1}{2}(\Delta\theta_1 + \Delta\theta_2)\,, \\ \alpha' M_4^2 &= \frac{Y^2}{4\pi^2\alpha'} + 1 - \frac{1}{2}(\Delta\theta_1 + \Delta\theta_2)\,, \end{aligned} \tag{2.9}$$

where Y denotes the separation between the branes in the third torus. This indicates that in any non-supersymmetric case one of these complex scalars will become tachyonic.

The tachyon that appears at the intersection will trigger the recombination of the branes. In the general case, the branes will recombine to only one brane wrapping the minimal volume supersymmetric 2-cycle in the total homology class [26]. Nevertheless, as we will explain in the last part of this section, there are some cases in which the final state can be a supersymmetric configuration with still two branes intersecting at angles.

Now that we have explained the general features of the model we have in mind, we must point out some relevant considerations that should be done about the model: first of all note that the R-R vanishing tadpole conditions must be satisfied in order for the theory to be well defined. When we have branes intersecting at angles on a compactified space, the R-R tadpole vanishing conditions are equivalent to the vanishing of the total homology charge of the system. In our configuration this is only fulfilled in the special case that we have a brane and an antibrane. In the general case we will need an extra brane with opposite total homology charge for the conditions to be satisfied. However this is not important for our analysis as we can place an additional brane at a large distance in the transverse directions, so it would not be involved in the dynamical evolution of the above mentioned brane configuration.

Another important assumption that we will make along this paper is that the moduli associated to the compact space and the dilaton are stabilized by some unknown mechanism. In other words, we will consider that the evolution of the closed string modes is much slower than the evolution of the open string modes. That means that we will only consider the attractive potential between the branes due to the interbrane distance Y as the relevant potential, which will be identified with the inflaton potential. This is the strongest assumption we will make and it is consistent as soon as the overall size of the compact space is larger compared with the distance between the branes.

Apart from the contribution coming from the R-R tadpole conditions we have to consider also the contribution coming from the NS-NS tadpoles. When a brane configuration is supersymmetric we have (from the closed string modes) that the contribution due to the NS-NS tadpoles (those that came from the dilaton plus graviton interactions) cancel exactly the contribution coming from the R-R tadpoles. If we break the supersymmetry of the system we will have that the attractive potential coming from the dilaton and graviton interactions will be greater than the repulsive potential due to the fact that both branes have same sign R-R charge. As a result, the branes will attract to each other. Then, in this case, since the supersymmetry of the system is being broken, there are going to be uncancelled NS-NS tadpoles that will act as a potential between the branes. This potential has been calculated in the previous subsection for the most general case, on a non-compact space. When dealing with compact spaces the expression (2.4) is modified in the following way

$$\mathcal{A}(Y,\Delta\theta_j) = \frac{V_p L^{4-n} F(\Delta\theta_j)}{2^{p-2}(2\pi^2\alpha')^{(p+1-n)/2}} \sum_{\omega_k \in Z} \int_0^\infty t^{-\frac{p+n-5}{2}} \exp^{-\frac{t\sum_k (Y_k + 2\pi\omega_k R)^2}{2\pi^2\alpha'}} dt\,, \tag{2.10}$$

where ω_k represents the winding modes of the strings on the directions transverse to the branes.[2] Nevertheless, if the distance between the branes is small compared with the compactification radii ($Y \ll (2\pi R)$), the winding modes would be too massive and then will not contribute to the low energy regime. That is, it will cost a lot of energy to the strings to wind around the compact space. If we translate this assumption to (2.10), the dominant mode will be the zero mode, and the potential can be written as in (2.6), (2.7), taking into account that we focus on the case where the number of angles is $n = 2$. In this case the potential (when normalised over the non-compact directions) for branes of different dimensions is just given by

$$V_{Dp}(Y,\Delta\theta_j) \;=\; -\frac{(2\pi R)^{(p-5)} F(\Delta\theta_j)}{2^{p-2}(2\pi^2\alpha')^{p-3}} \Gamma\left(\frac{5-p}{2}\right) Y^{(p-5)}\,, \tag{2.11}$$

$$V_{D5}(Y,\Delta\theta_j) \;=\; \frac{F(\Delta\theta_j)}{(4\pi^2\alpha')^2} \ln\frac{Y}{\Lambda_c}\,, \tag{2.12}$$

where the $(2\pi R)^{p-5}$ factor arises from the dimensions in which the branes become parallel on the compact dimensions (remember that R denotes the radius of the torus).

In the case of a two angled configuration the function $F(\Delta\theta_j)$ given in (2.5) can be written as

$$F(\Delta\theta_1,\Delta\theta_2) = \frac{(\cos\Delta\theta_1 - \cos\Delta\theta_2)^2}{\sin\Delta\theta_1 \sin\Delta\theta_2}\,. \tag{2.13}$$

[2] That means that the summation over k in (2.10) has only one term in the D6-brane case and it will be $Y_9 = |x_9^{(1)} - x_9^{(2)}|$ (see figure 1). In the D5-brane case we will have two terms: $Y_8 = |x_8^{(1)} - x_8^{(2)}|$ and $Y_9 = |x_9^{(1)} - x_9^{(2)}|$. Also in both cases we will denote $Y = \sqrt{\sum_k Y_k^2}$.

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Note that this expression has the right behaviour in the supersymmetric cases since it vanishes when $\Delta\theta_1 = \pm\Delta\theta_2$, as expected. In the limit $\Delta\theta_1 = \pm\Delta\theta_2 + \Delta\theta$, where the supersymmetry is slightly broken, that is $\Delta\theta \ll 1$, we find that $F(\Delta\theta_1, \Delta\theta_2) \simeq (\Delta\theta)^2$.

Apart from this contribution to the total potential of the system one should also consider the energy density of the branes that wrap a cycle over some of the compact dimensions. This energy density is given by the volume spanned by the branes over the compact directions times the tension of the branes. Then, the energy density of our two brane system is given by

$$E_0 = E_1 + E_2 = T_p(A_1 + A_2) = T_p A_T \,, \tag{2.14}$$

where T_p denotes the tension of the Dp-brane and is given by $T_p = M_s^{p+1}/g_s(2\pi)^p$, being g_s the string coupling and M_s the string mass. Also by A_i we denote the "volume" spanned by the Dp_i-brane along the compactified directions[3]

$$A_i = (2\pi R)^{p-3} \sqrt{\left(\left(n_1^{(i)}\right)^2 + \left(m_1^{(i)}\right)^2\right)\left(\left(n_2^{(i)}\right)^2 + \left(m_2^{(i)}\right)^2\right)}. \tag{2.15}$$

Note that if the configuration is supersymmetric the energy (2.14) does not vanish, as one would expect. This is because we are not considering the total energy of the system. As we explained in the previous subsection we need to introduce an additional brane to cancel the R-R tadpoles. If we take into account this additional brane the total energy will cancel (2.14) in the supersymmetric case.

At the end of the day, the total potential of the system is given by

$$V_T = E_0 + V_{Dp}(\Delta\theta_I, Y) \,, \tag{2.16}$$

where the exact form of $V_{Dp}(\Delta\theta_I, Y)$ depends on the dimension of the branes that are present in the model, as it is shown in (2.11) and (2.12). Recall that in (2.16) we consider that all the moduli are stabilized except for the distance Y between the branes, that will play the role of the inflaton field.

2.3 Brane recombination

In this subsection we consider the evolution of the system described before. Then we will be dealing with two D5 or D6-branes with a four-dimensional intersection over the non-compact dimensions and making two angles over the compact dimensions (as shown in figure 1). The angles at which the branes intersect each other, as can be seen in (2.8), are given in terms of $(n_I^{(i)}, m_I^{(i)})$, which denote the number of times that the Dp_i-brane wraps a cycle on the I'-th torus. As we mentioned before this configuration is supersymmetric when $\Delta\theta_1 \pm \Delta\theta_2 = 0$, and preserves, in that case, 1/4 of the supersymmetries.

Then, if (a_i, b_i) denote the homology cycles of the torus, the homology class of each Dp_i-brane is given by the expression

$$[\Pi_{Dp_i}] = \left(n_1^{(i)}[a_1] + m_1^{(i)}[b_1]\right) \otimes \left(n_2^{(i)}[a_2] + m_2^{(i)}[b_2]\right), \tag{2.17}$$

[3]We will be considering for simplicity that in the D6-brane case, $n_{(1)}^{(3)} = n_{(2)}^{(3)} = 1$, and $m_{(1)}^{(3)} = m_{(2)}^{(3)} = 0$, so that the branes are separated in the x_9 direction.

being the total homology class of the configuration the sum of the homology class of each brane

$$[\Pi_T] = [\Pi_{Dp_1}] + [\Pi_{Dp_2}]\,. \tag{2.18}$$

When $\Delta\theta_1 \pm \Delta\theta_2 \neq 0$ the configuration is non supersymmetric and one of the complex scalars in (2.9) becomes tachyonic triggering the recombination of the system. Since the branes intersect at angles over the compact space, the system will recombine into another that wraps a minimal volume supersymmetric cycle (that is, an special lagrangian) over the compact dimensions. This cycle must have the same total homology class than the initial configuration,[4] being this final configuration a minimum of the tachyon potential.

As is shown in (2.14) the energy of the initial configuration is given in terms of the wrapping numbers of both initial branes. Nevertheless, since this is non supersymmetric such configuration will not have a minimum energy or, in other words, it will not saturate the BPS bound for the energy of a system in the same homology class. The BPS bound for a configuration of two Dp-branes intersecting at two angles on a $T^2 \times T^2$ can be calculated in terms of the initial energy of each brane [26]. The exact expression for this BPS bound is given by

$$E_{BPS} = \sqrt{(E_1 + E_2)^2 - 4E_1E_2 \sin^2 \frac{\Delta\theta}{2}}\,, \tag{2.19}$$

where $\Delta\theta = \Delta\theta_1 - \Delta\theta_2$, and E_1, E_2, are just the energy of each brane in the initial configuration. Note that when $\Delta\theta = 0$ the configuration is supersymmetric and so we recover the energy (2.14) of the initial configuration, as one would expect.

If we are consider configurations very close to the supersymmetric case, then $\Delta\theta \ll 1$. Expanding (2.19) around $\Delta\theta = 0$ we get

$$E_f \simeq E_1 + E_2 - \frac{E_1E_2}{2(E_1 + E_2)}(\Delta\theta)^2\,. \tag{2.20}$$

The variation of the tachyonic potential is related to the difference of the energy density between the initial and final configurations [27], and in this case it is just given by

$$E_0 - E_f \simeq \frac{E_1E_2}{2(E_1 + E_2)}(\Delta\theta)^2\,. \tag{2.21}$$

As we see, the difference is proportional to the supersymmetry breaking parameter $\Delta\theta$. This means that when $\Delta\theta = 0$ the difference of energy between both configurations vanishes, as expected if one takes into account that the supersymmetric case is stable and therefore there are no tachyons.

The expression (2.19) gives us information about the energy of the final configuration, but not about the cycle that it wraps. In fact, in the general case, after the recombination of the branes one will end up with just one brane with energy (2.19) that wraps a supersymmetric non-factorisable 2-cycle over the compact space [26].

[4] Remember that this has to be fulfilled in order for the RR-tadpoles to be cancelled.

Nevertheless this is not the only possible final configuration. As we will now show it is interesting to note that for a large class of initial configurations there is also the possibility to end, after the recombination of the system, with again branes intersecting at two angles, but in a supersymmetric configuration. This is an interesting possibility since intersecting branes can have chiral matter at their intersections and then are potentially useful to construct realistic models of particle physics.

To analyze this possibility one has to take into account several facts. The first thing is that for this recombination to occur, one needs a final configuration that wraps also a factorisable 2-cycle with the same total homology class as the initial one, which is given by (2.18). That is, the final configuration must satisfy

$$\begin{aligned} [\Pi_T] = &\left(n_{f1}^{(1)}[a_1] + m_{f1}^{(1)}[b_1]\right) \otimes \left(n_{f2}^{(1)}[a_2] + m_{f2}^{(1)}[b_2]\right) + \\ &+ \left(n_{f1}^{(2)}[a_1] + m_{f1}^{(2)}[b_1]\right) \otimes \left(n_{f2}^{(2)}[a_2] + m_{f2}^{(2)}[b_2]\right), \end{aligned} \tag{2.22}$$

where $(n_{fI}^{(i)}, m_{fI}^{(i)})$ denotes the wrapping numbers of the Dp_i-brane over the homology cycles of the I'-th torus in the final configuration. Written for each homology cycle the eq. (2.22) tells us that the relation between the initial and final wrapping numbers is the following

$$\begin{aligned} n_1^{(1)}n_2^{(1)} + n_1^{(2)}n_2^{(2)} &= n_{f1}^{(1)}n_{f2}^{(1)} + n_{f1}^{(2)}n_{f2}^{(2)}, \\ n_1^{(1)}m_2^{(1)} + n_1^{(2)}m_2^{(2)} &= n_{f1}^{(1)}m_{f2}^{(1)} + n_{f1}^{(2)}m_{f2}^{(2)}, \\ m_1^{(1)}n_2^{(1)} + m_1^{(2)}n_2^{(2)} &= m_{f1}^{(1)}n_{f2}^{(1)} + m_{f1}^{(2)}n_{f2}^{(2)}, \\ m_1^{(1)}m_2^{(1)} + m_1^{(2)}m_2^{(2)} &= m_{f1}^{(1)}m_{f2}^{(1)} + m_{f1}^{(2)}m_{f2}^{(2)}. \end{aligned} \tag{2.23}$$

The second thing that we should take into account is that the energy of the final configuration must saturate the BPS bound (2.19). Nevertheless, if the final configuration has two branes intersecting at two angles, then its final energy must be given by

$$\begin{aligned} E_f = T_p(2\pi R)^{p-3}\Bigg[&\sqrt{\left(\left(n_{f1}^{(1)}\right)^2 + \left(m_{f1}^{(1)}\right)^2\right)\left(\left(n_{f2}^{(1)}\right)^2 + \left(m_{f2}^{(1)}\right)^2\right)} + \\ &+ \sqrt{\left(\left(n_{f1}^{(2)}\right)^2 + \left(m_{f1}^{(2)}\right)^2\right)\left(\left(n_{f2}^{(2)}\right)^2 + \left(m_{f2}^{(2)}\right)^2\right)}\Bigg]. \end{aligned} \tag{2.24}$$

This means that in order for the final configuration to saturate the BPS bound the expression for the energy (2.24) must be equal to the expression (2.19). To see if that is possible it is useful to rewrite (2.19) as

$$E_{BPS} = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\Delta\theta}\,, \tag{2.25}$$

using that $\sin^2\Delta\theta/2 = \frac{1}{2}(1-\cos\Delta\theta)$. Now one can write $\cos\Delta\theta$ in terms of the wrapping numbers of the initial configuration by means of (2.8). With that and also writing E_1 and

E_2 as in (2.15) one finally gets

$$E_{BPS} = T_p(2\pi R)^{p-3}\left[\left(n_1^{(1)}n_2^{(1)} + n_1^{(2)}n_2^{(2)}\right)^2 + \left(m_1^{(1)}m_2^{(1)} + m_1^{(2)}m_2^{(2)}\right)^2 + \right.$$
$$\left. + \left(n_1^{(1)}m_2^{(1)} + n_2^{(2)}m_1^{(2)}\right)^2 + \left(n_2^{(1)}m_1^{(1)} + n_1^{(2)}m_2^{(2)}\right)^2\right]^{1/2} . \qquad (2.26)$$

Furthermore, as the final configuration must be supersymmetric and we are not interested in final configurations with parallel branes, the only non-trivial solution is given by configurations that satisfy

$$\begin{aligned}\left(n_{f1}^{(1)}, m_{f1}^{(1)}\right) &= \left(n_{f2}^{(2)}, m_{f2}^{(2)}\right) , \\ \left(n_{f2}^{(1)}, m_{f2}^{(1)}\right) &= \left(n_{f1}^{(2)}, m_{f1}^{(2)}\right) .\end{aligned} \qquad (2.27)$$

Replacing this into (2.24) the energy of the final configuration will be given by

$$E_f = T_p(2\pi R)^{p-3}\sqrt{\left(2n_{f1}^{(1)}n_{f2}^{(1)}\right)^2 + \left(2n_{f1}^{(1)}m_{f2}^{(1)}\right)^2 + \left(2m_{f1}^{(1)}n_{f2}^{(1)}\right)^2 + \left(2m_{f1}^{(1)}m_{f2}^{(1)}\right)^2} . \qquad (2.28)$$

Now inserting the relations (2.27) for the final homological charges in (2.23) the conditions for the homological charges become

$$\begin{aligned} n_1^{(1)}n_2^{(1)} + n_1^{(2)}n_2^{(2)} &= 2n_{f1}^{(1)}n_{f2}^{(1)} , \\ n_1^{(1)}m_2^{(1)} + n_1^{(2)}m_2^{(2)} &= m_1^{(1)}n_2^{(1)} + m_1^{(2)}n_2^{(2)} = n_{f1}^{(1)}m_{f2}^{(1)} + m_{f1}^{(1)}n_{f2}^{(1)} , \\ m_1^{(1)}m_2^{(1)} + m_1^{(2)}m_2^{(2)} &= 2m_{f1}^{(1)}m_{f2}^{(1)} . \end{aligned} \qquad (2.29)$$

Finally, just by using (2.29) and comparing (2.28) with (2.26) it is straight forward to see that when

$$\mid n_1^{(2)}m_2^{(2)} - n_2^{(2)}m_1^{(2)} \mid = \mid n_{f2}^{(1)}m_{f1}^{(1)} - n_{f1}^{(1)}m_{f2}^{(1)} \mid , \qquad (2.30)$$

the energy of the final configuration saturates the BPS bound. Note also that the condition (2.30) is never zero since we are considering that the initial configuration has always branes intersecting at two angles. That means that we will only have final configurations with branes at angles, not with parallel ones.

Then we can conclude that if we begin with an initial configuration that satisfies

$$\begin{aligned} \left(n_1^{(1)}n_2^{(1)}+n_1^{(2)}n_2^{(2)}\right)\left(m_1^{(1)}m_2^{(1)}+m_1^{(2)}m_2^{(2)}\right) &= \left(n_1^{(1)}m_2^{(1)}+n_2^{(2)}m_1^{(2)}\right)\left(m_1^{(1)}n_2^{(1)}+n_1^{(2)}m_2^{(2)}\right) , \\ n_1^{(1)}m_2^{(1)} + n_1^{(2)}m_2^{(2)} &= m_1^{(1)}n_2^{(1)} + m_1^{(2)}n_2^{(2)} , \end{aligned} \qquad (2.31)$$

then the system will recombine into again two branes making two angles.

One can also see that the final angle between the branes after the recombination is just given by

$$\mid \tan \Delta\theta_f \mid = \frac{2|n_1^{(2)}m_2^{(2)} - n_2^{(2)}m_1^{(2)}|}{n_1^{(1)}n_2^{(1)} + n_1^{(2)}n_2^{(2)} + m_1^{(1)}m_2^{(1)} + m_1^{(2)}m_2^{(2)}} \qquad (2.32)$$

and also from here, since $n_1^{(2)}m_2^{(2)} \neq n_2^{(2)}m_1^{(2)}$, we see that in these cases we will not have parallel branes after recombination, as we mentioned previously.

An example of such a recombination process is shown in figure 2, in which we have taken two branes intersecting in $T^2_{(1)}$ with wrapping numbers $(1,0)$ (green solid line), $(0,1)$ (red dashed line), respectively, and in $T^2_{(2)}$ with $(2,1)$, $(1,0)$. This brane configuration can be recombined to give a configuration with again two branes at angles but with wrapping numbers $(1,0)$, $(1,1)$ on the first torus and $(1,1)(1,0)$ on the second one, respectively. Note that the angles that make the branes in both tori are the same so the final configuration preserves $1/4$ of the supersymmetries.

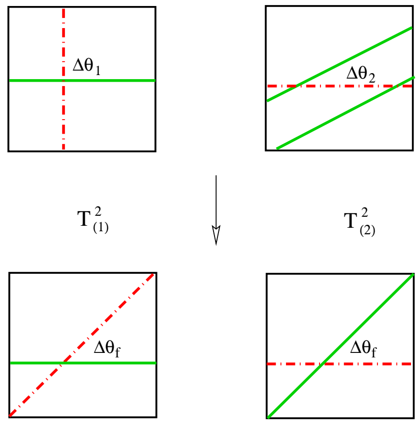

Figure 2: Example of a concrete configuration that recombines again into intersecting branes.

Summarizing, we have the following result: as we know, when the tachyon appears, the branes will recombine into another brane configuration that wraps a cycle that minimizes the energy. In the general case the final brane configuration is only one brane wrapping a non-factorisable 2-cycle on the compact space. However, for some special cases the final configuration that minimizes the energy contains branes at angles but in a supersymmetric configuration (that is, intersecting at the same angle in both tori).

We remark here again that this fact might have interesting consequences from the point of view of building realistic models from intersecting branes. Nevertheless we must point that in order to obtain a chiral spectrum from these configurations one should introduce extra objects in the model, such as for example orientifolds in the spirit of the supersymmetric models described in [25].

3. Angled brane inflation

In this section we discuss the inflationary parameters derived from the potential between the branes at angles discussed in the previous section. This potential will be valid as soon as the distance between the branes is larger than the string scale, l_s, and small compared with the radius of the torus, as the potential was derived under these assumptions. When the distance becomes too small, (see (2.9)) the system will develop tachyonic modes giving rise to a natural end of inflation via extra fields (in this case tachyonic fields) as in the hybrid inflationary models. This type of inflationary scenarios have been discussed in string brane models for various configurations [3, 8, 9, 11]. This kind of mechanism to end inflation (thorough the tachyonic field) seems to be generic and it happens also in our present cases, as one might have expected.

3.1 Relevant inflationary parameters

We consider now the relevant inflationary parameters that any model of inflation should satisfy, and will study if the constraints coming from those parameters are consistent with the approximations we have done. That is, we will study if it is possible to obtain a successful inflationary epoch within these brane models intersecting at two angles.

The four-dimensional effective action for the distance between the branes, Y, can be written in the following form

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}M_P^2\,\mathcal{R}-\frac{1}{4}T_p\,A_T(\partial_\mu Y)^2+V(Y)\right], \qquad (3.1)$$

where $\mathcal{R}$ is the Ricci scalar in 4 dimensions, $M_P^2 = M_s^2 V_6\, g_s^{-2} = M_s^2(2\pi R M_s)^6\, g_s^{-2} = 2.4\times 10^{18}$ GeV is the four-dimensional Planck mass, R is the radius of each square torus and V_6 is the total volume of the compact dimensions. Then, the canonically normalised field associated to the brane separation, Y, is given by

$$\Psi = Y\sqrt{\frac{T_p A_T}{2}} = (Y M_s)M_s\sqrt{\frac{M_s^{p-3}A_T}{2\,g_s\,(2\pi)^p}}\,. \qquad (3.2)$$

Note that since $A_T \propto R^{p-3}$ the normalised field Ψ has dimensions of mass, as it should.

Let us now recall the standard equations of motion of a Friedmann-Robertson-Walker universe with a scalar field. These are given by

$$\begin{aligned}\ddot{\Psi}\,+\,3H\dot{\Psi}\,&=\,-\frac{dV}{d\Psi}\,,\\ H^2\,&=\,\frac{1}{3M_p^2}\left[V+\frac{\dot{\Psi}^2}{2}\right],\end{aligned} \qquad (3.3)$$

where H is the Hubble parameter. The slow roll conditions require that $|\ddot{\Psi}| \ll 3H|\dot{\Psi}|$ and $\dot{\Psi}^2 \ll V$, i.e. that the friction and potential terms dominate. From these conditions one can derive the two slow-roll parameters

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = M_P^2\frac{V''}{V}\,, \qquad (3.4)$$

which should be small in accordance with the field equations, so that $|\epsilon| \ll 1$ and $|\eta| \ll 1$ for slow roll inflation to occur. The primes above denote derivatives with respect to the inflaton field Ψ.

The number of e-foldings occurring after the scales probed by the COBE data leave the horizon can be computed as [29]

$$N=\int H dt = \frac{1}{M_P^2}\int_{\Psi_{\rm end}}^{\Psi_*}\frac{V(\Psi)}{V'(\Psi)}d\Psi\,, \qquad (3.5)$$

where $\Psi_* \propto Y_*$ is the brane separation when the primordial density perturbation exits the de Sitter horizon during inflation. Since in this model the end of inflation is not given by the cease of the slow-roll conditions in what follows, we will assume that $Y_* \gg Y_{\rm end}$. This is a common assumption in hybrid inflationary models and we will justify it later.

The amplitude of the density perturbation when it re-enters the horizon, as observed by Cosmic Microwave Background (CMB) experiments is given by:

$$\delta_H = \frac{2}{5}\mathcal{P}_{\mathcal{R}}^{1/2} = \frac{1}{5\pi\sqrt{3}}\frac{V^{3/2}}{M_p^3\,V'} = 1.91\times 10^{-5}\,, \qquad (3.6)$$

where the value of δ_H is implied by the COBE results [28].

The spectral index and its derivative[5] can be expressed in terms of the slow roll parameters [29], and they are given by

$$n-1=\frac{\partial\ln\mathcal{P}_{\mathcal{R}}}{\partial\ln k}\simeq 2\eta-6\epsilon\,,\qquad \frac{dn}{d\ln k}\simeq 24\epsilon^2-16\epsilon\eta+2\xi^2\,. \tag{3.7}$$

where $\xi^2\equiv M_P^2\frac{V'V'''}{V^2}$.

The gravitational wave spectrum can be calculated in a similar form as for the scalar one, and the gravitational spectral index $n_{\rm grav}$ is given by

$$n_{\rm grav}=\frac{d\ln\mathcal{P}_{\rm grav}(k)}{d\ln k}=-2\epsilon\,. \tag{3.8}$$

As we will see later, in both the cases we will consider, the variation of the spectral index and the gravitational perturbations will be negligible, since the slow roll parameter, ϵ, turns out to be too small, $\epsilon\leq 0.01$.

With these parameters we can now look separately at the two models of branes intersecting at two angles to determine if they can give us a successful inflationary period.

3.2 D5-brane model

Consider now type-IIB string theory compactified on a squared, factorisable, six-dimensional torus T^6, as explained in section 2. We now introduce two D5-branes which span the four dimensional, $\mathbb{R}^{3,1}$, non-compact dimensions and intersect at two angles, $\Delta\theta_I$, between them in two of the tori, as seen in figure 1. Intersecting brane models of this type have been considered in [23].

The effective potential for the canonically normalised field Ψ (or Y) is obtained by combining the tension of the branes and the interbrane potential energy (see eq. (2.16)). For the pair of D5-branes intersecting at two angles in the compact space, and choosing an appropriate cutoff in (2.12), it is given by

$$V=T_5A_T+\frac{M_s^4F(\Delta\theta_I)}{(2\pi)^4}\ln\left[\frac{\Psi}{M_s}\right], \tag{3.9}$$

with $T_5\,A_T=(M_s^2\,A_T)M_s^4/g_s(2\pi)^5$; the index $I=1,2$ denotes the torus in which the branes make an angle and from (2.13) we have that,[6] $F\simeq(\Delta\theta)^2$.

With this potential (3.9), the slow roll parameters become

$$\epsilon\simeq\frac{(2\pi)^2g_s^2F^2}{2(M_s^2A_T)^2}\left(\frac{M_P}{\Psi}\right)^2,\qquad \eta\simeq-\frac{2\pi g_sF}{(M_s^2A_T)}\left(\frac{M_P}{\Psi}\right)^2. \tag{3.10}$$

Then it is clear that $|\epsilon|<|\eta|$, so that it would be enough to consider η in the rest of the calculations. So in order for inflation to occur, we only need that the ratio of F to $M_s^2A_T$

[5] In order to study the scale dependence of the spectrum, whatever its form is, one can define an effective *spectral index* $n(k)$ as $n(k)-1\equiv d\ln\mathcal{P}_{\mathcal{R}}/d\ln k$. This is equivalent to the power-law behaviour that one assumes when defining the spectral index as $\mathcal{P}_{\mathcal{R}}(k)\propto k^{n-1}$ over an interval of k where $n(k)$ is constant. One can then work $n(k)$ and its derivative by using the slow roll conditions defined above.

[6] Recall that we are breaking slightly the supersymmetry so $\Delta\theta_1=\Delta\theta_2+\Delta\theta$ with $\Delta\theta\ll 1$.

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be small enough. That is, a successful inflationary period will take place if

$$\frac{F}{M_s^2 A_T} \ll 1 . \tag{3.11}$$

Notice that this requirement is very easy to achieve without requiring a fine tuning of the parameters, since the volume A_T can be done very large (see (2.15)), and also F can be made very small. The number of e-folds can be calculated from (3.5) to give

$$N \simeq \frac{(M_s^2 A_T)}{2g_s(2\pi)F}\left(\frac{\Psi_*}{M_P}\right)^2 , \tag{3.12}$$

where Ψ_* is proportional to the separation between the branes when primordial density perturbation leaves the horizon during inflation. Then, if (3.11) holds there would be enough number of e-foldings for inflation to take place. In terms of the number of e-folds, the slow roll parameters and the spectral index can be written in the form

$$\epsilon \simeq \frac{1}{4\,N^2}\frac{\Psi_*^2}{M_P^2} , \qquad \eta \simeq -\frac{1}{2\,N} , \qquad n \simeq 1 - \frac{1}{N} ; \tag{3.13}$$

which is well within the present bounds from the CMB anisotropies for $N \gtrsim 40$.

Now we have to impose the COBE normalisation, that is, the amplitude of the scalar perturbations (3.6) give us

$$\delta_H \simeq \frac{2^{1/2}\,(M_s^2 A_T)\,(M_s/M_P)^2\,N^{1/2}}{5\pi\sqrt{3}\,(2\pi)^3\,g_s\,F^{1/2}} = 1.91 \times 10^{-5} ; \tag{3.14}$$

and using the expression for M_P we have from here that

$$F^{1/2} = \frac{2^{1/2} N^{1/2} g_s (M_s^2 A_T)}{5\pi\sqrt{3}\,(2\pi)^3\,(1.91 \times 10^{-5})(2\pi R M_s)^6} . \tag{3.15}$$

The distance between the branes can be calculated using (3.2), (3.12) and (3.15). This gives us

$$M_s\,Y_* \simeq \frac{2\sqrt{2}\,g_s\,N}{5\pi\sqrt{3}(1.91 \times 10^{-5})\,V_6^{1/2}} . \tag{3.16}$$

Note that it depends only on the overall compactification volume, V_6.

Now, if we take $N \sim 60$, with a weak string coupling of $g_s = 0.1$ and a compactification radius of say, $2\pi R M_s = 25$, we can compute the angle difference and the separation of the branes at the beginning of inflation. From (3.16) one gets a size for the separation between the branes of $M_s\,Y_* = 2.1 = 0.08(2\pi R M_s)$, and then consistent with our approximations.

Also for the angular difference $\Delta\theta$ we get

$$\Delta\theta \sim 3.5 \times 10^{-8}\, M_s^2 A_T , \tag{3.17}$$

then the angle difference can be made larger using the volume A_T, that is, making bigger the volume that the branes wrap over the compactified dimensions. For example, for a volume of order $M_s^2 A_T \sim 3 \times 10^4$, the difference between the angles will be $\Delta\theta \sim 10^{-3}$.

The spectral index is then $n \sim 0.98$ and its variation negligible as well as the gravitational waves spectrum. The string scale in this case is $M_s = M_P g_s/(2\pi R M_s)^3 \sim 1.54 \times 10^{13}$ GeV, that is smaller than M_P as one would expect since we are considering weakly coupled string theory. These numbers are quite common in models of inflation and we end up with a successful inflationary scenario for an angular difference not so fine tuned, and it can be enlarged using A_T (as it is shown in (3.17)). The end of inflation will be discussed in the next section, however, as already mentioned, the appearance of the tachyon will trigger the end of inflation as in any hybrid inflationary model coming from non-stable brane configurations.

3.3 D6-brane model

Let us now consider type-IIA theory on $\mathbb{R}^{3,1} \times T^6$ with the six torus factorisable, as discussed previously. We now introduce two D6-branes which intersect at two angles in the first two tori, but they are separated in the last torus by a distance Y (see figure 1). Intersecting models using D6-branes have been studied widely in both non supersymmetric [18, 19, 20, 22] and supersymmetric [25] versions.

The effective potential for this model is given by (2.16) in terms of the distance between the branes Y. Using (3.2) we can rewrite it in terms of the inflaton field Ψ and it becomes

$$V(Y, \Delta\theta_I) = T_6 A_T + M_s^4 \frac{V_6^{1/6}\, F(\Delta\theta_I)}{(2\pi)^3} \sqrt{\frac{2\pi g_s}{M_s^3 A_T}} \left(\frac{\Psi}{M_s}\right). \tag{3.18}$$

Here again $F \simeq (\Delta\theta)^2$ and $V_6^{1/6} = (2\pi R M_s)$. The slow roll parameters become very simple in this case, and in particular, notice that $\eta = 0$. So we have,

$$\epsilon \simeq \frac{F^2\, (2\pi)^7\, g_s^3 V_6^{1/3}}{2\, (M_s^3\, A_T)^3} \left(\frac{M_P}{M_s}\right)^2. \tag{3.19}$$

From here it is clear that in order to have an inflationary period, the following condition has to be satisfied,

$$\frac{F^2}{(M_s^3\, A_T)^3} \ll 1\,, \tag{3.20}$$

which again it is easy to get since the volume A_T may be very large and also F very small. The number of e-folds can be calculated using (3.5) to give

$$N \simeq \left(\frac{M_s}{M_P}\right)^2 \frac{(M_s^3\, A_T)^{3/2}}{(2\pi)^{7/2} g_s^{3/2}\, V_6^{1/6}\, F} \frac{\Psi_*}{M_s}\,. \tag{3.21}$$

which will be big enough for inflation to occur as soon as the constraint (3.20) is satisfied. In terms of the number of e-folds, the slow roll parameter and the spectral index can be written in the form

$$\epsilon \simeq \frac{1}{2\, N^2} \frac{\Psi_*^2}{M_P^2}\,, \qquad n \simeq 1 - \frac{3\Psi_*^2}{N^2\, M_P^2} \sim 1\,. \tag{3.22}$$

Since ϵ is very small, in this case the spectral index is almost scale invariant and so its variation is going to be negligible.

The amplitude of the scalar perturbations (3.6) give us

$$\delta_H \simeq \frac{(M_s/M_P)^3 (M_s^3\, A_T)^2}{5\pi\sqrt{3}(2\pi)^6\sqrt{2\pi} g_s^2 V_6^{1/6} F} . \tag{3.23}$$

From here we can then compute the angle to be,

$$F = \frac{g_s\, (M_s^3\, A_T)^2}{5\pi\sqrt{3}(2\pi)^6\sqrt{2\pi}(1.91\times 10^{-5})(2\pi R\, M_s)^{10}} . \tag{3.24}$$

The distance between the branes is found to be

$$M_s\, Y_* \simeq \frac{\sqrt{2} N g_s}{5\pi\sqrt{3}(1.91\times 10^{-5}) V_6^{1/2}} . \tag{3.25}$$

Note that this distance depends only on the square root of the total compactified volume and not on the volume that the branes wrap, as in the previous D5-brane case (see eq. (3.16)). In fact it is easy to see that this is a general feature in models with potentials of the form $V = T_p A_T + BY^d$, with $d \in \mathbb{Z}$, or $V = T_p A_T + B \ln Y$.

In order to get some numbers, consider again $N \sim 60$ and a radius of $2\pi R M_s \sim 25$, with g_s as before, the distance between the branes is $M_s\, Y_* = 2 = 0.08(2\pi R M_s)$ which is consistent with our approximations. Also for this values one can easily compute the angular difference,

$$\Delta\theta \sim 1.3\times 10^{-9} \left(M_s^3 A_T\right) , \tag{3.26}$$

then if we take a volume of order $M_s^3 A_T \sim 8\times 10^5$, will give us $\Delta\theta \sim 10^{-3}$ and again it can be increased using a larger volume. Note that for this values of the parameters the string scale is of the same order as in the five brane case. Also for this values $\epsilon \sim 10^{-12}$ and then the gravitational perturbations will be again too small, so we do not consider them here.

We have then seen that also in this simple case the potential between the branes can give rise to an inflationary regime within string motivated scenarios. The important issue which arises at this point is the end of inflation, that will be triggered, as was explained in previous sections, by a tachyonic field. We will discuss this point briefly in the next section.

4. End of inflation

The inflaton field Ψ is proportional to the distance between the branes Y, as can be seen in (3.2). Since the potential of the system is attractive we will have that the inflaton field Ψ rolls towards smaller values during inflation. Nevertheless, inflation does not end when the slow roll conditions ceased to be satisfied (i.e. $\epsilon \sim 1$, $\eta \sim 1$). The reason is that when the distance between the branes becomes small compared with the string scale, the configuration develops tachyonic modes and many more degrees of freedom apart from the interbrane distance become relevant [30]. This, as was first pointed out in [3], provides a nice realization of a hybrid inflationary scenario [17].

As explained in the first section this tachyonic fields will trigger the recombination of the system. In general the final configuration will be one brane wrapping a special lagrangian cycle over the compact space. Nevertheless, as we mentioned previously, for

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a large class of initial configurations the recombination of the system will be such that it will allow for the final configuration to be also branes at angles but in a supersymmetric configuration (that is, intersecting at the same angle in both tori). This means that also after inflation we will have a configuration with intersecting branes and then suitable for building realistic models of particle physics. Moreover as the volume that the branes wrap in the initial configuration is not fixed by inflationary constraints (that is, it is not fixed in order for the relevant inflationary parameters to be satisfied), this means that one can begin with a configuration that allows a suitable recombination.

These systems develop a negative mode when the distance between the branes in the compactified directions becomes smaller than a critical value given in terms of the angles at which the branes intersect. This critical value parametrises the end of inflation and, as one can see from eqs. (2.9), it is given by $M_s Y_{\rm end} = \sqrt{(2\pi^2)\Delta\theta}$. Then when $M_s Y \lesssim M_s Y_{\rm end}$ tachyonic open strings connecting both branes will appear. For our models, $\Delta\theta \sim 10^{-3}$ and so, this value is given by $Y_{\rm end} \sim 0.14$.

It has been realised that the tachyon potential by itself does not produce slow-roll inflation [3], but its importance arises when inflation ends. In [31] it has been shown that the roll along the tachyonic direction is very fast, and the initial potential energy is quickly converted in gradient energy for the tachyon field. The precise details of the reheating process are out of the scope of the present letter,[7] however we can give a crude approximation for the temperature of reheating. If we assume that all of the initial vacuum energy of the branes is converted into radiation, an estimate can be done by considering T_{RH}^4 as the initial potential energy [3], giving

$$T_{RH} \sim M_s \left[\frac{M_s^{p-3} A_T}{g_s (2\pi)^{p+1}}\right]^{1/4} . \tag{4.1}$$

If we introduce here the values for the volume spanned by the branes used in the cases considered, the D5 and D6-brane models, using $M_s \sim 10^{13}$ GeV and $g_s \sim 0.1$ we get a reheating temperature of order $T_{RH} \sim 10^{13}$ GeV.

As we already mentioned, when the branes collide a tachyon appears and the system recombine. Generically lower dimensional D$(p-2k)$-branes can be formed during this process [30, 33]. Since this happens after inflation it is important to check whether monopole-like objects or domain walls can be formed or not, as the presence of such objects may destroy nucleosynthesis or over close the universe. Nevertheless, as was pointed in [12, 34], this does not seem to happen on these brane configurations. The main reason is the fact that the size of the compactified dimensions is much smaller that the Hubble radius H. Then the only topological defects that can form cosmologically by the Kibble mechanism whose production will not be heavily suppressed are those of codimension two, where the codimension should lie in the uncompactified space. That means that only cosmic string-like defects are likely to be produced in the four-dimensional space. The cosmic strings produced in this way are in general D$(p-2)$-branes with $(p-3)$ dimensions over the compact space, spanning the same compctified volume as the former Dp-brane, that we

[7] For a more detailed discussion on tachyonic reheating in this kind of models, see for example [32].

will denote by A_p (as defined in (2.15)). Then, following the idea of [34] we can calculate the tension of a cosmic string produced in this way to be

$$\mu = T_{p-2} A_p = \frac{M_s^{p-1}}{(2\pi)^{p-2} g_s} A_p \,. \tag{4.2}$$

Gravitational interactions of the cosmic strings are given in terms of the dimensionless parameter $G\mu$, where G is the Newton's constant. Using (4.2) we can write this parameter as

$$G\mu = \left(\frac{M_s}{M_P}\right)^2 \frac{M_s^{p-3} A_p}{(2\pi)^{p-2} g_s} = \frac{g_s M_s^{p-3} A_p}{V_6 (2\pi)^{p-2}} \,. \tag{4.3}$$

This value must satisfy $G\mu \lesssim 10^{-6}$ in order not be in conflict with present observations [28]. From here we can estimate an upper bound for the compactified "volume" of the Dp-branes A_p. This must satisfy

$$M_s^{p-3} A_p \lesssim 10^5 - 10^6 \,. \tag{4.4}$$

This value is within the range of volumes that we have used in the previuos section. We should point out that this is only an estimation and that a deeper analysis of the defect production and reheating after inflation in the present scenario would be very interesting.

5. Conclusions

Along this paper we have considered interesting models of branes intersecting at angles in a compact six-dimensional space. The detailed configuration that we have studied is type-II string theory on $\mathbb{R}^{3,1} \times T^6$ where the compact space is a six-dimensional torus factorisable in three two-dimensional tori. We introduced two Dp-branes ($p = 5, 6$) in our configuration in such a way that they intersected at angles on two of the tori, but allowing them to be separated a distance Y on the remaining one. These configurations are interesting since they may be relevant in the construction of realistic — intersecting at angles — brane models (that is, models with a spectrum close to the one of the standard model of particle physics). Such models have been studied, for example, in [18]–[24]. We have considered two interesting cases, one in type IIA and the other one in type-IIB string theory.

Like in the case of branes intersecting at just one angle [11, 12], we have found, in the two cases considered, that inflation is generic when the configuration is slightly non supersymmetric. These models give a successful period of inflation, consistent with observations. Moreover we have found a remarkable feature when considering two angled configurations in contrast with the one angled models. In fact, the configurations we have considered, open up the possibility of interesting new recombinations of the branes after inflation. In previous models [11, 12], recombination at the end of inflation gave rise to the formation of only one brane wrapping some supersymmetric cycle in the compact space. However, if one wants to build realistic models of particle physics after inflation, those models turned may not be very attractive, since chiral matter would not be present at the end of inflation.

We have shown how one can improve this situation by considering branes intersecting at two angles. In this case, there is the possibility of ending up with only one brane wrapping a supersymmetric non-factorisable 2-cycle in the compact space after inflation. Nevertheless there also exist the possibility that the final configuration be that with two branes intersecting at angles in the compact space, but in a supersymmetric way, i.e. making the same angle in both tori. This possibility is quite attractive since there are a number of realistic models constructed from intersecting branes. There are also some supersymmetric versions with intersecting branes already in the literature [25]. In any case, one should note that independently of the type of recombination of the branes after inflation we obtain, within these string D-brane models, a successful inflationary epoch consistent with the present observations.

Further investigation on the non-trivial recombinations of the branes is worth to be done and also we must point out that this recombinations may be useful in other aspects of string theory. Another thing we want to remark is the fact that along this kind of models one assumes that many moduli are fixed. It would be interesting to study how to stabilize these moduli in a controlled way. A possibility may be the introduction of fluxes in the configurations so one can use the techniques developed, for example, in [35, 36] to fix some of the moduli. Finally, we mention again that a deeper investigation on the real process of reheating and defect production after inflation is definitely worth to be done.

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